



Measuring energy equipartition in globular clusters

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Abstract. Globular clusters (GCs) evolve toward a state of partial energy equipartition over their long-term evolution, due to relaxation processes that induce exchange of energy between stars. This establishes a mass-dependent internal kinematics: more-massive stars lose kinetic energy and sink toward the center while less-massive stars gain kinetic energy and move outward. In this contribution, we show how the mass-dependence of the velocity dispersion can be efficiently described, both in simulations and observations, with an exponential function characterized by one parameter indicating how close to full energy equipartition a system is. Moreover, we show that the degree of equipartition reached by a GC tightly correlates with its dynamical state and therefore we propose this equipartition-dynamical state relation as a tool to characterize the relaxation condition of a cluster, based on kinematic measurements.

Key words. Stars: kinematics and dynamics – Globular clusters: general

1. Introduction

Despite the apparent simplicity of Galactic globular clusters (GCs), their formation and evolution is still poorly understood. Their properties that we see today are the result of the combined effects of internal astrophysical processes (two-body relaxation and stellar evolution) and external processes (interaction with the Milky Way), that took place over their long term > 10 Gyr evolution. The deep understanding of the current dynamical properties of GCs is therefore fundamental to shed light onto the earliest phases of their formation.

Since GCs are old stellar systems, two-body interactions between stars have been effi-

cient in shaping their internal structure. In particular, GCs evolve towards a state of thermalization, where stars with different masses approach the same energy (Spitzer 1987). This is known as energy equipartition: massive stars lose kinematic energy sinking into the center of the cluster, while, vice versa, low-mass stars gain kinetic energy and move to the outer regions. Even if GCs are not expected to reach full energy equipartition (e.g., Spitzer 1969; Trenti & van der Marel 2013; Bianchini et al. 2016a; Spera et al. 2016) this can still produce a significant mass dependence of the kinematics. This additional complication in the dynamical interpretation of GCs has been often neglected due to the lack of kinematic data

able to sample velocities for stars with different masses, other than the massive-bright giants with masses of $0.8 - 0.9 M_{\odot}$.

However, the mass-dependent kinematics is now within reach of our observational capabilities, thanks to the combination of traditional spectroscopic-based line-of-sight velocities, high-precision Hubble Space Telescope (HST) proper motions, and state-of-the-art IFU observations with MUSE at the Very Large Telescope. While traditional line-of-sight samples provide velocity measurements for giant stars only, HST proper motions and MUSE line-of-sight measurements allow us to collect large samples ($> 10\,000$) of velocities for both giant stars and less-massive main sequence stars (see HSTPROMO data sets for 22 GCs, Bellini et al. 2014; Watkins et al. 2015a,b; Bianchini et al. 2016b; Baldwin et al. 2016 and MUSE data from Kamann et al. 2016), allowing for the first time for a systematic study of the kinematics down to $\approx 0.3 - 0.4 M_{\odot}$.

Driven by the advancement of our observational capabilities, in this contribution (based on the results presented in Bianchini et al. 2016a) we introduce a new approach to describe the mass dependence of the kinematics in GCs, suitable for both simulations and observations. Using this approach we are able to quantify the degree of energy equipartition reached by a cluster and to study the variety of mass-dependence of kinematics that we could expect for the Galactic GC system.

2. Fitting the mass dependence of kinematics

We consider a set of Monte Carlo cluster simulations, developed by Downing et al. (2010) with the Monte Carlo code of Giersz (1998) (see also Hypki & Giersz 2013). The simulations are evolved in quasi-isolation and include an initial mass function, stellar evolution, primordial binaries, and a relatively high number of particles, providing a realistic description of the long-term evolution of GCs with a single stellar population. We consider a total of 6 simulations with 500 000 initial particles, characterized by different concentrations and binary fractions and an additional simulation

with 2 000 000 particles (for details see Table 1 of Bianchini et al. 2016a). For each simulation we consider three time-snapshots at 4, 7, 11 Gyr and we construct the projected velocity dispersion profile as a function of stellar mass, $\sigma(m)$, restricted within the projected half-light radius. We consider projected quantities in order to enable a direct comparison with observations.

The fitting function that we propose is an exponential function, characterized by a velocity scale parameter σ_0 and one mass scale parameter m_{eq} :

$$\sigma(m) = \begin{cases} \sigma_0 \exp\left(-\frac{1}{2} \frac{m}{m_{\text{eq}}}\right) & \text{if } m \leq m_{\text{eq}}, \\ \sigma_{\text{eq}} \left(\frac{m}{m_{\text{eq}}}\right)^{-1/2} & \text{if } m > m_{\text{eq}}. \end{cases} \quad (1)$$

Here, σ_0 indicates the value of velocity dispersion at $m = 0$, while σ_{eq} corresponds to the value of velocity dispersion at m_{eq} . The parameter m_{eq} quantifies the degree of partial energy equipartition reached by the systems. For $m > m_{\text{eq}}$ the system is characterized by constant full energy equipartition ($\sigma \propto m^{-1/2}$). This function can be considered as an extension of the simple power-law usually assumed $\sigma \propto m^{-\eta}$ (Trenti & van der Marel 2013), that, however, is valid only locally.

We perform two fits to the velocity dispersion profile as a function of mass using Eq. 1: one using all the stars in the mass range $0.1 - 1.8 M_{\odot}$, and one restricting to only observable stars in the mass range $0.4 - 1.0 M_{\odot}$ (to match the typical observations). The fit to all the stars is performed to the binned profiles, while in the case restricted to observable stars only, we use a discrete fitting approach, particularly flexible for a future application to real data. Fig. 1 shows the results for one of our simulations. Our proposed exponential function provides an excellent fit to all our simulations, also when restricted to the stellar mass range available from observations, and gives as best fit parameter $m_{\text{eq}} \gtrsim 1.5 M_{\odot}$, indicating that all the stars sampled below this mass are characterized by a state of partial energy equipartition.

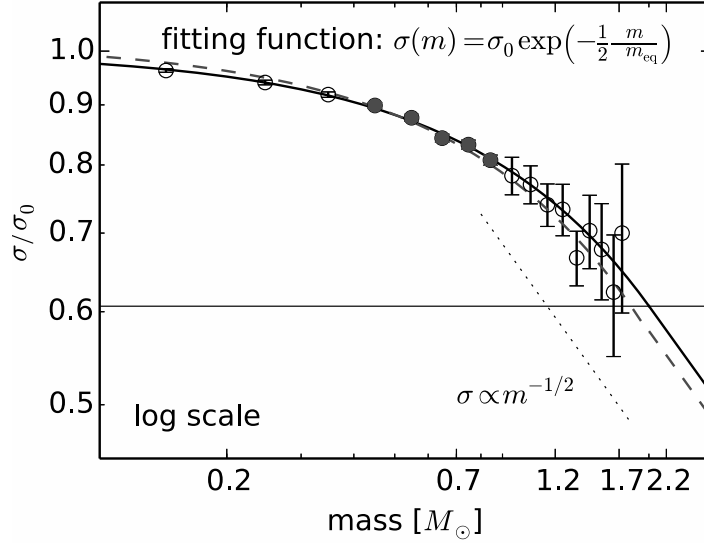


Fig. 1. Fit to the projected velocity dispersion (of one of our simulations) as a function of the stellar mass $\sigma(m)$ using the exponential fitting function introduced with Eq. 1. Two fits are performed: one to all the data in the mass range $0.1 - 1.8 M_{\odot}$ within the half-mass radius (empty circles/dashed line), and one restricted to the observable mass range $0.4 - 1.0 M_{\odot}$ (full circles/solid line). The profiles are normalized at $m = 0$, using the best fit parameter σ_0 (see Eq. 1) The dotted line shows the power-law function $\sigma \propto m^{-1/2}$ expected for full energy equipartition. The horizontal line intersects the fitting function at $m = m_{\text{eq}}$; beyond this mass the fitting function reaches constant full equipartition.

3. Measuring the dynamical state of a cluster

Having measured the degree of partial energy equipartition reached by our simulations using the exponential fitting function (Eq. 1), we can now correlate the values of m_{eq} obtained with GC properties. We introduce the quantity $n_{\text{rel}} = T_{\text{age}}/T_{\text{rc}}$, with T_{age} the age of the cluster and T_{rc} the core relaxation time calculated from Eq. 2 of Bianchini et al. (2016a). This quantity indicates the number of relaxation times that a cluster has experienced, with higher n_{rel} corresponding to more relaxed stellar systems. Fig. 2 shows a tight correlation between m_{eq} and n_{rel} , indicating that the onset of energy equipartition depends on the units of relaxation time experienced by a cluster.

This relation can provide a fundamental tool to measure the relaxation condition of a cluster: with a measure of $n_{\text{eq}} = T_{\text{age}}/T_{\text{rc}}$ is possible to predict the m_{eq} parameter, hence

the dynamics of dark stellar remnants or of stars with not easily measurable kinematics (see Baldwin et al. 2016 for an application to blue straggler stars, and Bianchini et al. 2016b for binary stars). Vice versa, with a kinematic measure of m_{eq} , one can predict the n_{rel} for a given cluster and characterize its relaxation condition, indicating at which stage of evolution the system is (in a complimentary and independent way to the dynamical clock introduced by Ferraro et al. 2012 based on the radial distribution of blue straggler stars).

4. Conclusions

In this contribution, we introduced a novel approach to measure the degree of partial energy equipartition reached by GCs, suitable for both simulations and observations, starting from realistic Monte Carlo cluster simulations.

We showed that an exponential fitting function provides an excellent fit to the mass-

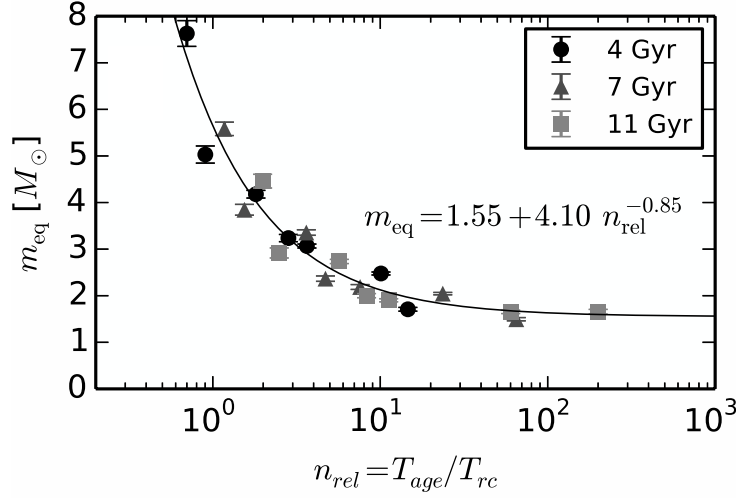


Fig. 2. Correlation between the parameter m_{eq} obtained from the fits to the simulations and the dynamical state of a cluster measured as $n_{\text{rel}} = T_{\text{age}}/T_{\text{rc}}$, with T_{age} the age of the cluster and T_{rc} core relaxation time. The plot shows that the level of energy equipartition reached by a cluster depends on its relaxation condition. Well relaxed clusters (characterized by $n_{\text{rel}} > 20$) reach a maximum value for the degree of energy equipartition. The solid line is the best fit for the $m_{\text{eq}} - n_{\text{rel}}$ correlation.

dependent velocity dispersion (induced by the onset of equipartition) both in the entire stellar mass range and in the observable stellar mass range. The fitted parameter m_{eq} quantifies how close to full energy equipartition a system is. Note that our function can be considered an extension of the commonly used power-law function $\sigma \propto m^{-\eta}$ that is instead only valid for restricted mass ranges.

We find that the degree of partial energy equipartition reached by a GC tightly correlates with its dynamical state. This tight relation can serve as a tool to investigate the dynamical condition of a GC: given a relaxation state, it is possible to predict the m_{eq} parameter, and therefore the mass-dependence of the kinematics. Vice versa, measuring the equipartition parameter m_{eq} from kinematics, it is possible to retrieve the internal dynamical state of a cluster. Finally, we anticipate an application of our novel approach to state-of-the-art line-of-sight and proper motion data sets.

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